

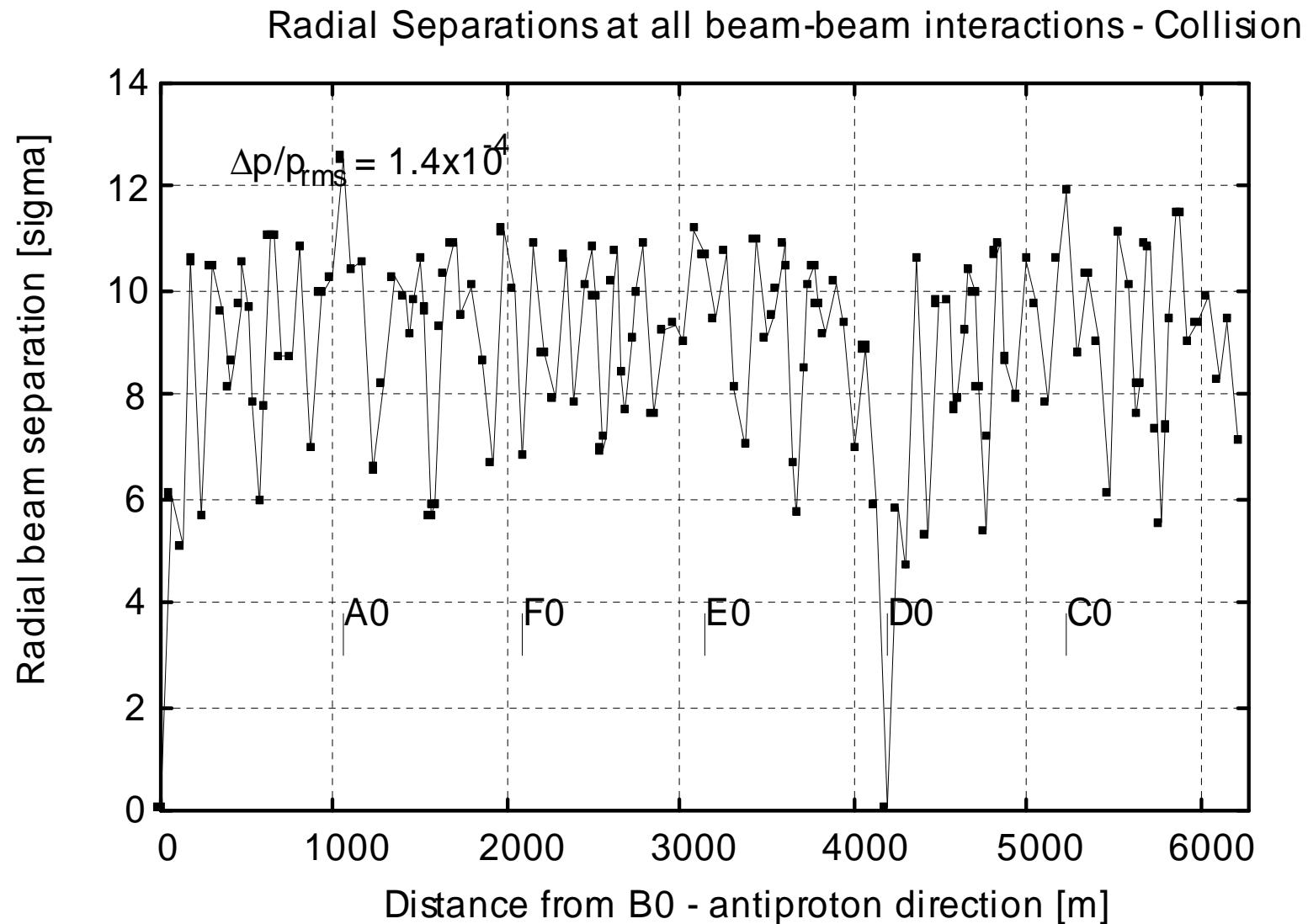
## **Beam-Beam Compensation with Wires**

- Theory gives requirements for ideal correction of one round well separated long-range beam-beam interaction by one wire:
  - Phase difference multiples of  $\pi$  in both planes
  - Ratio of horizontal and vertical beta functions at location of beam-beam interaction and wire the same
  - Current\*length of wire related to bunch intensity by  $ecN_b$
- Not easy to asses analytically what happens when these conditions are not satisfied exactly

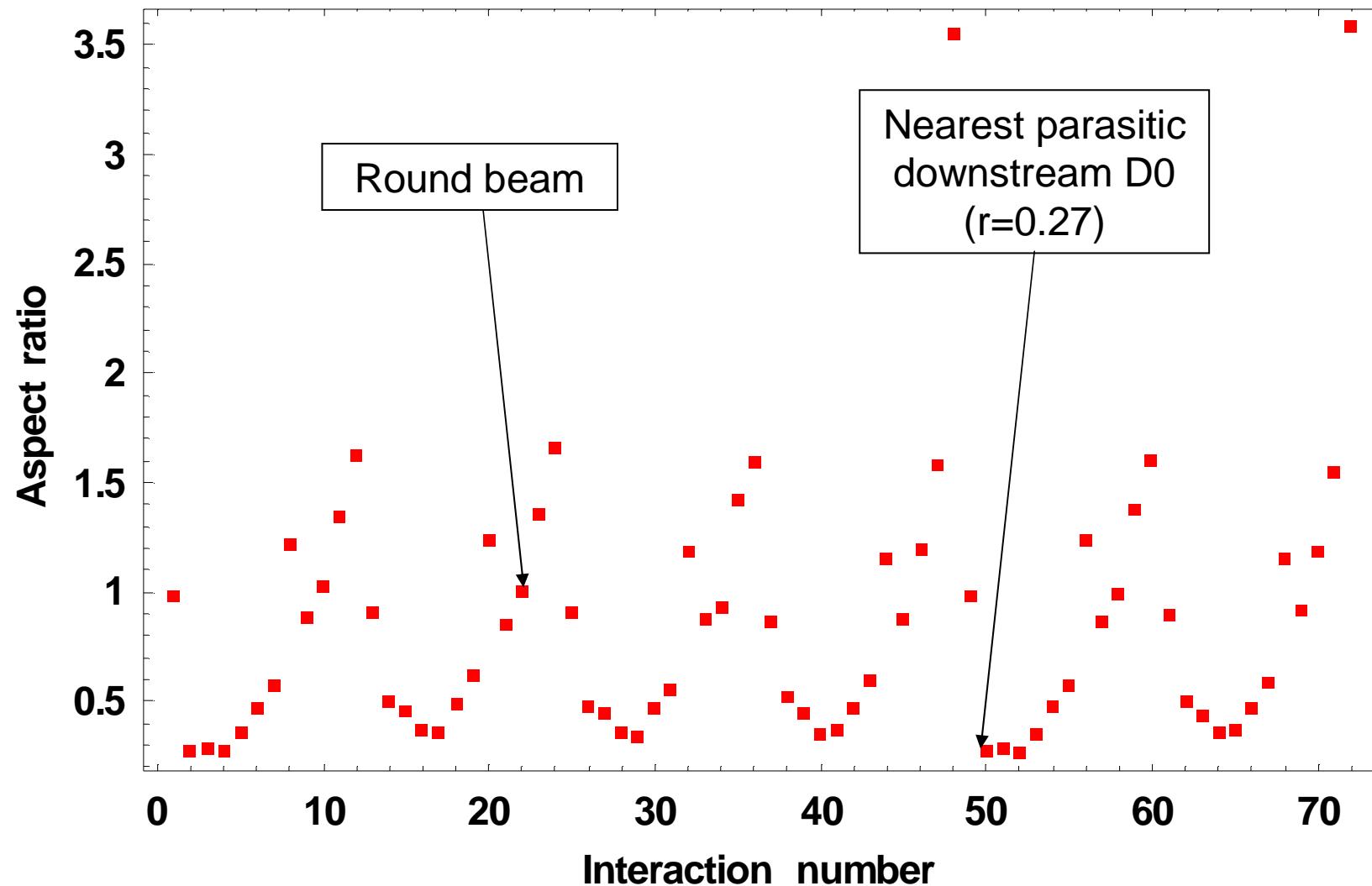
# Departure from the ideal conditions

- In the Tevatron (even without lattice nonlinearities), the requirements cannot be satisfied exactly, because:
  - In general beams are not round
  - Finite separation (  $5\sigma < \text{separation} < 13\sigma$  at collision )
  - The location of the  $\pi$  phase separation in the horizontal and vertical planes do not coincide longitudinally
  - At the longitudinal position where the phase differences come close to the required values, the ratio of the beta functions are not the same as at the beam-beam interaction's location
- Moreover, there are 72 (injection) /70 (collision)/ long-range beam-beam interactions spread all over the ring's circumference and in phase space
- Impossible to employ 72 or even more wires

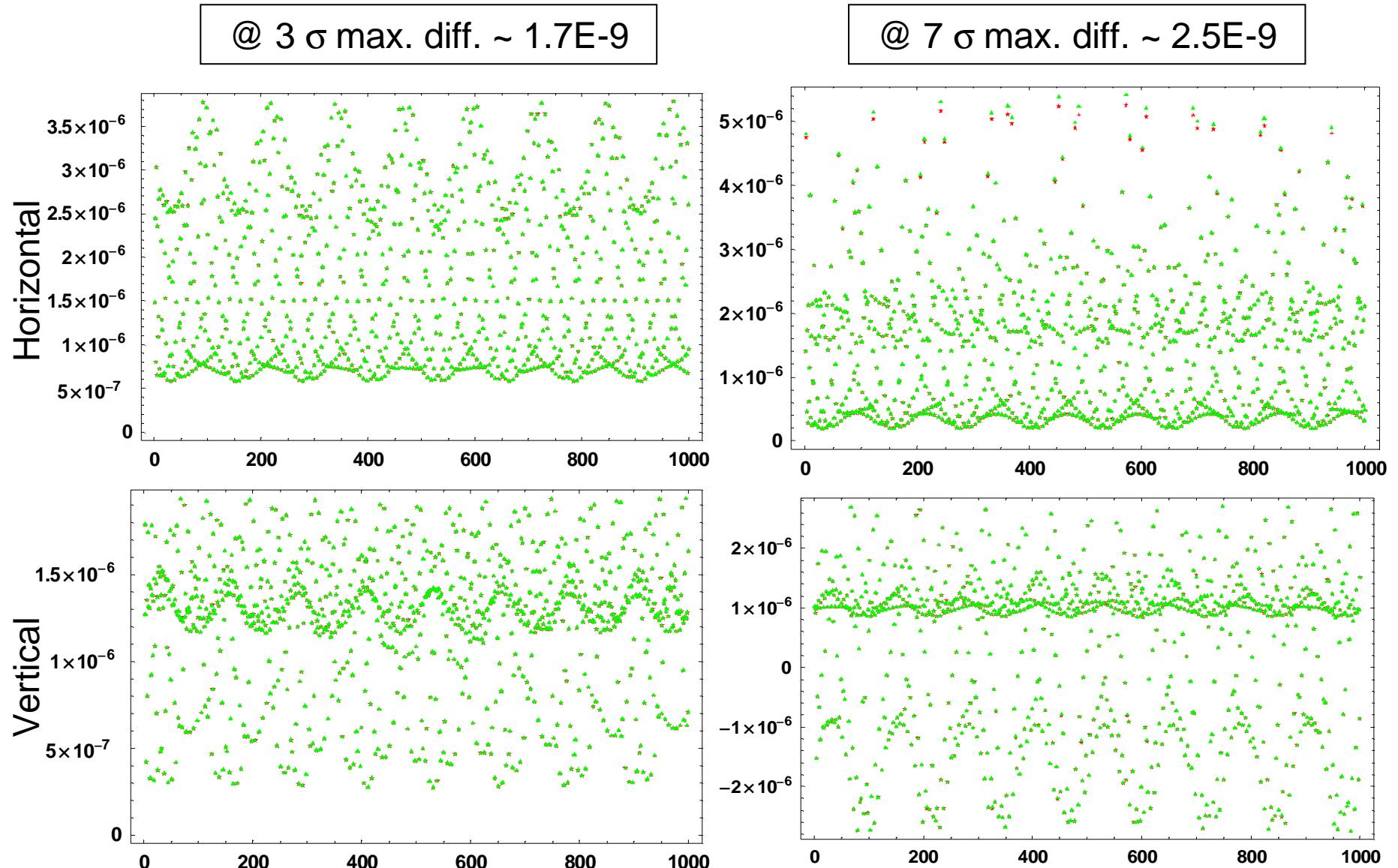
# Separations at collision



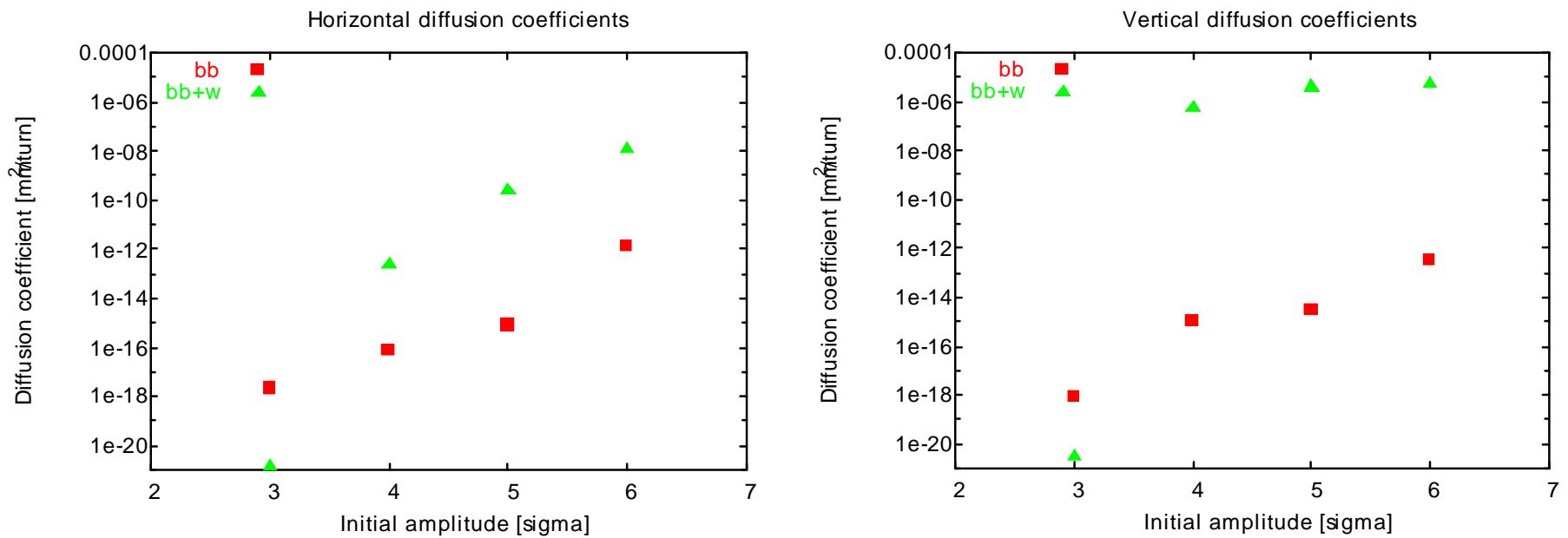
# Proton beam aspect ratios at collision



# **Kicks felt by a particle due to the round interaction and the wire placed to correct locally over the first $10^3$ turns**

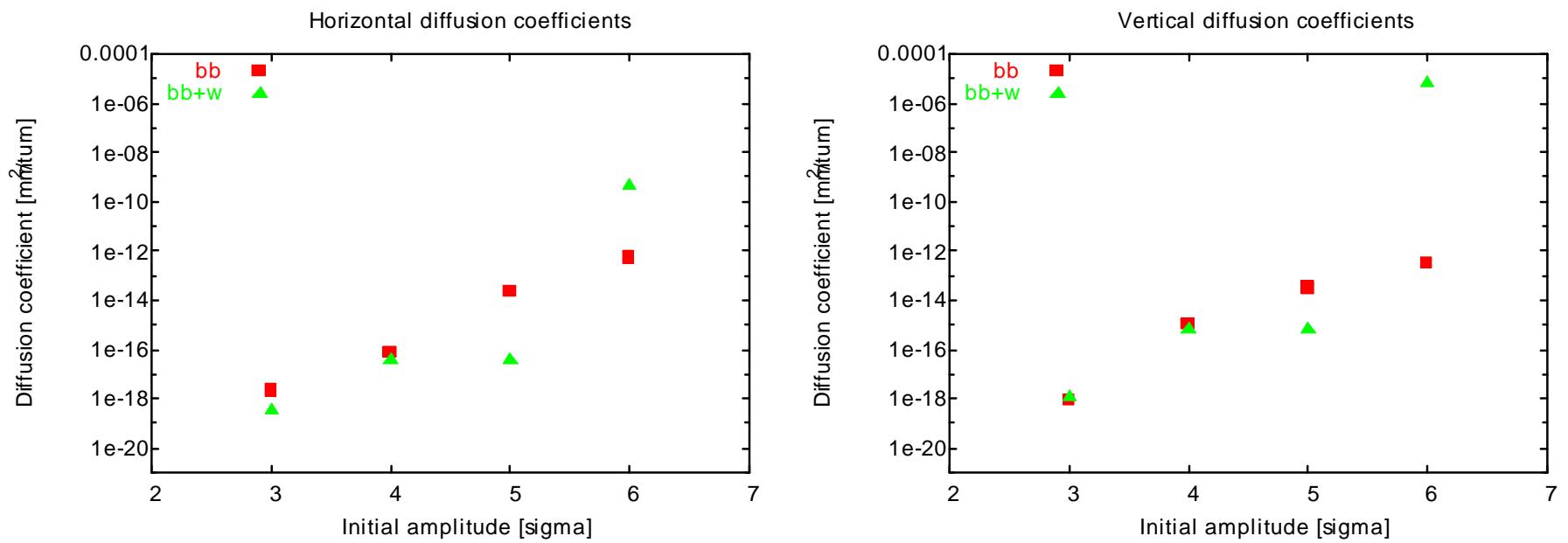


# Diffusion coefficients for local correction of the round beam-beam



Separation at the location of the parasitic is  $\sim 6.6 \sigma$

# Diffusion coefficients for local correction of the round beam-beam with separation increased by 50 %



Artificially increased the separation at the location of the parasitic to  $\sim 10 \sigma$

Clear sign of the importance of the exponential part in the round beam-beam kick.

# First order perturbation theory of close-to-round beam-beam kicks

Assume that  $\sigma_x > \sigma_y$  and  $x, y > 0$ .

$$\Delta x' = \frac{N_b r_p}{\gamma_p} \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}} \operatorname{Im} F_1(x, y) \quad \Delta y' = \frac{N_b r_p}{\gamma_p} \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}} \operatorname{Re} F_1(x, y)$$

$$F_1(x, y) = W\left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right) - \exp\left(-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2}\right) W\left(\frac{\frac{\sigma_y}{\sigma_x}x + i\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}}\right)$$

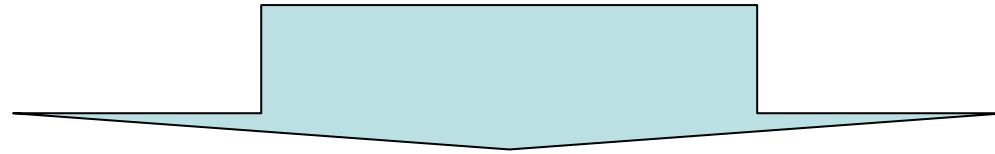
If  $\sigma_x \sim \sigma_y$  than can expand  $W(z)$  around  $|z| \rightarrow \infty$  to first order in  $1/z$

$$W(z) \underset{|z| \rightarrow \infty}{\propto} e^{-z^2} + \frac{i}{\sqrt{\pi z}} \left( 1 + \frac{1}{2z^2} + \frac{3}{4z^4} + \dots \right)$$

# First order perturbation theory of close-to-round beam-beam kicks

$$\Delta x' = \frac{2N_b r_p}{\gamma_p} x \left[ \frac{1}{x^2 + y^2} - \sigma_y^2 \frac{\sigma_x \sigma_y}{\sigma_y^4 x^2 + \sigma_x^4 y^2} \exp\left(-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2}\right) \right]$$

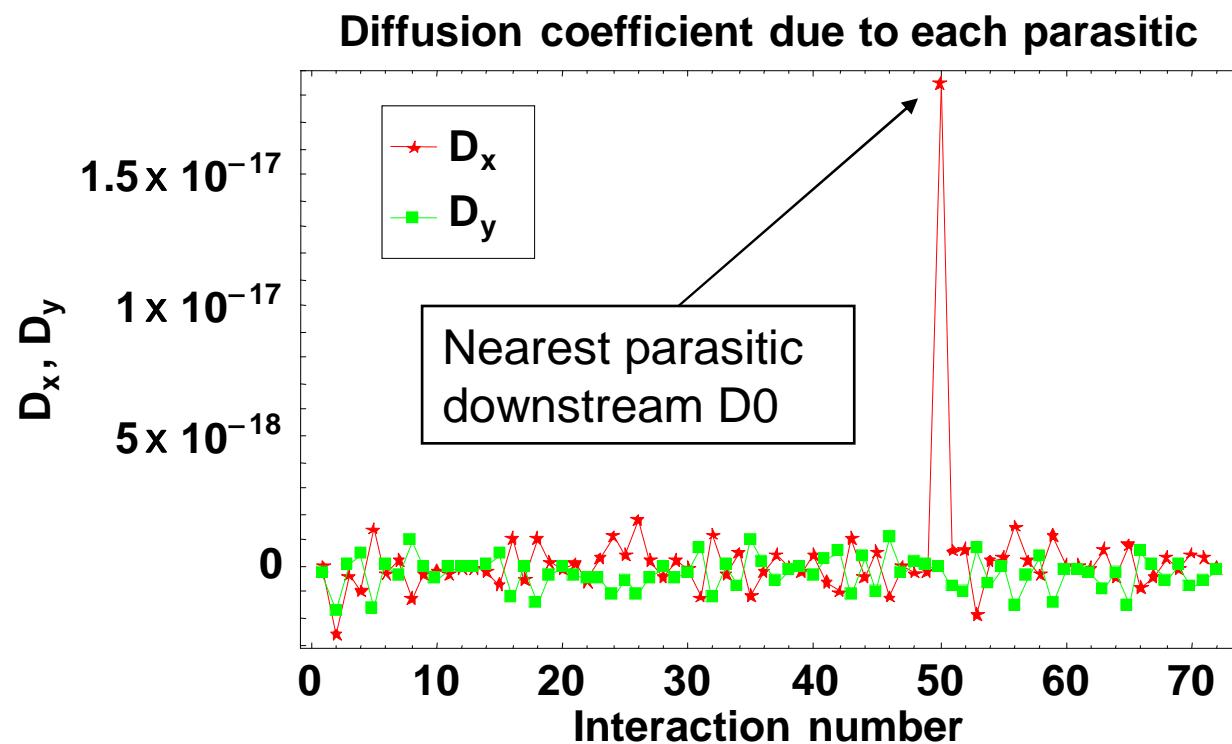
$$\Delta y' = \frac{2N_b r_p}{\gamma_p} y \left[ \frac{1}{x^2 + y^2} - \sigma_x^2 \frac{\sigma_x \sigma_y}{\sigma_y^4 x^2 + \sigma_x^4 y^2} \exp\left(-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2}\right) \right]$$



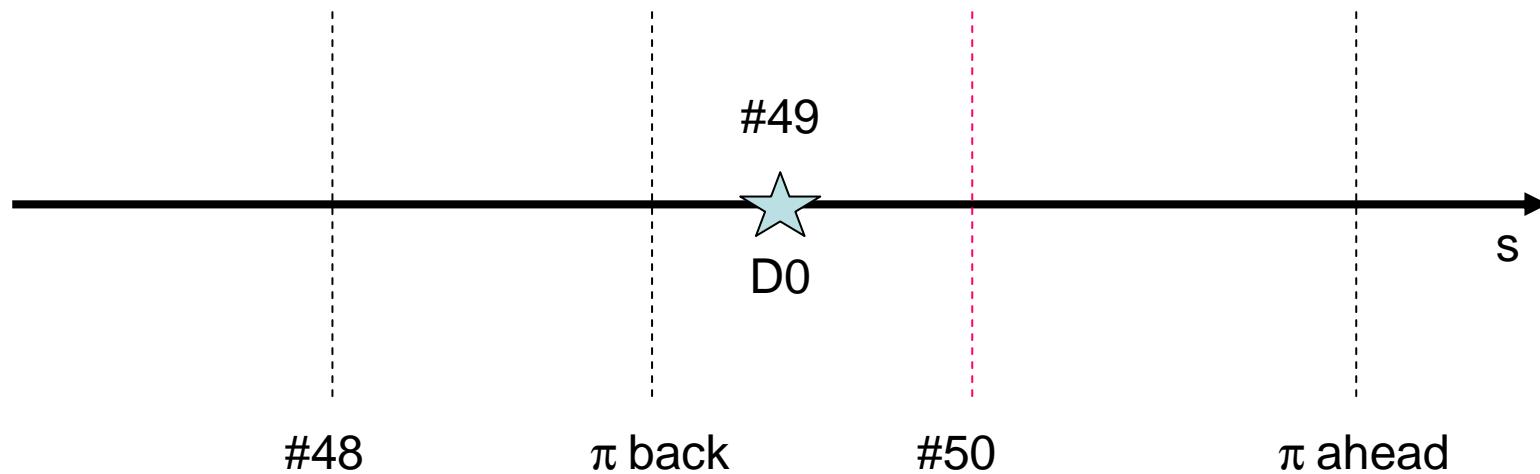
***The far-kick region coincides with the round beam case, and the near-kick region has the same falloff with the radial amplitude as the round beam case. Therefore, the wire compensation should be robust against small errors in optics and/or wire placement/current.***

# Diffusion coefficient of each parasitic

Coefficient calculated using 200 randomly chosen particles  
with  $5\sigma$  initial amplitudes after  $10^6$  turns

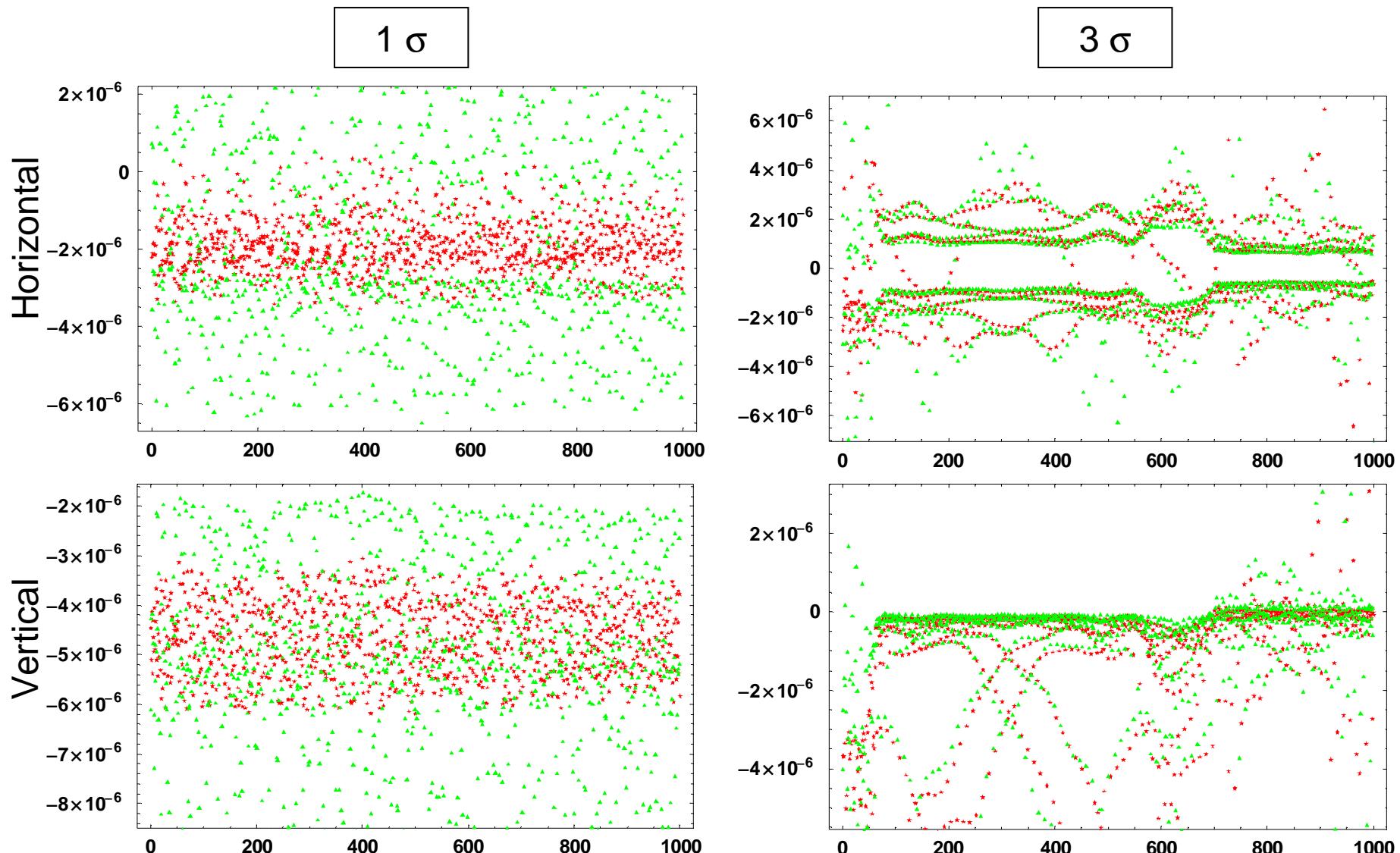


# Schematic of wire placement

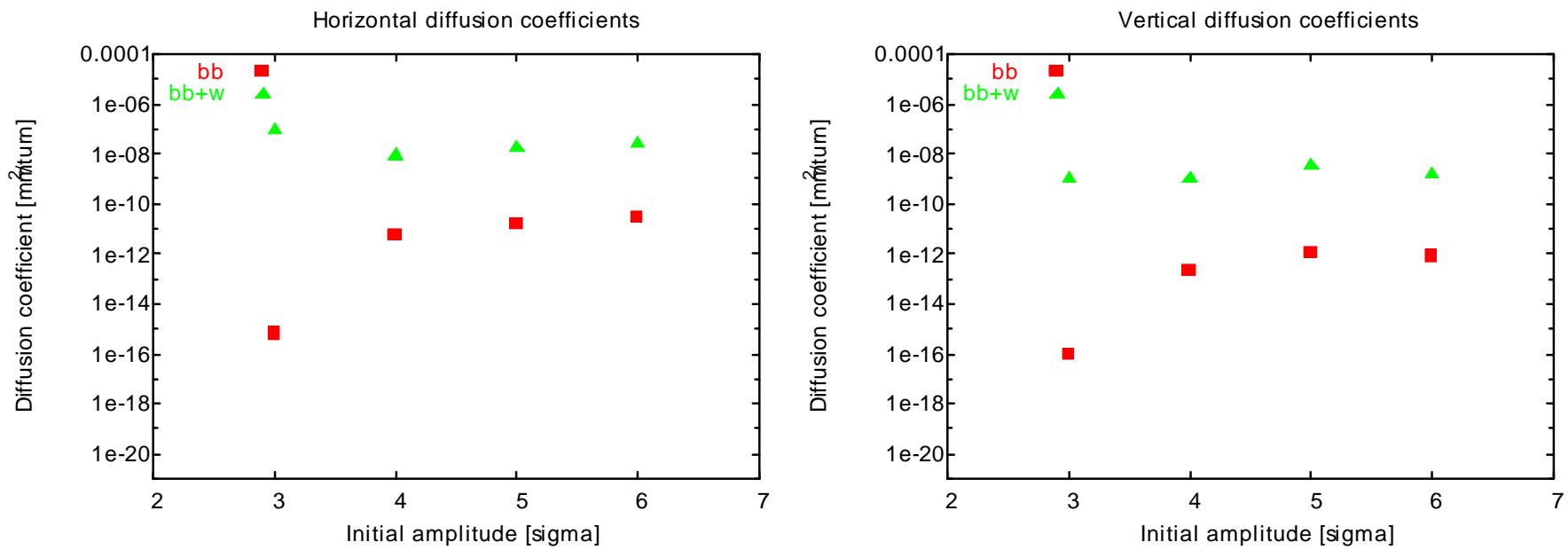


Pbar direction

# Kicks felt by a particle at different amplitudes from the **parasitic** and **wire** over $10^3$ turns

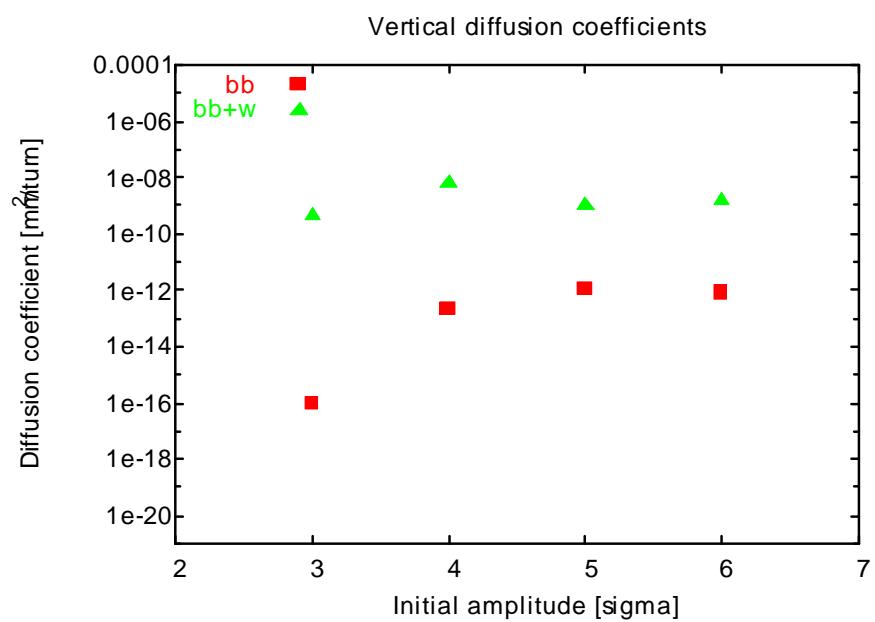
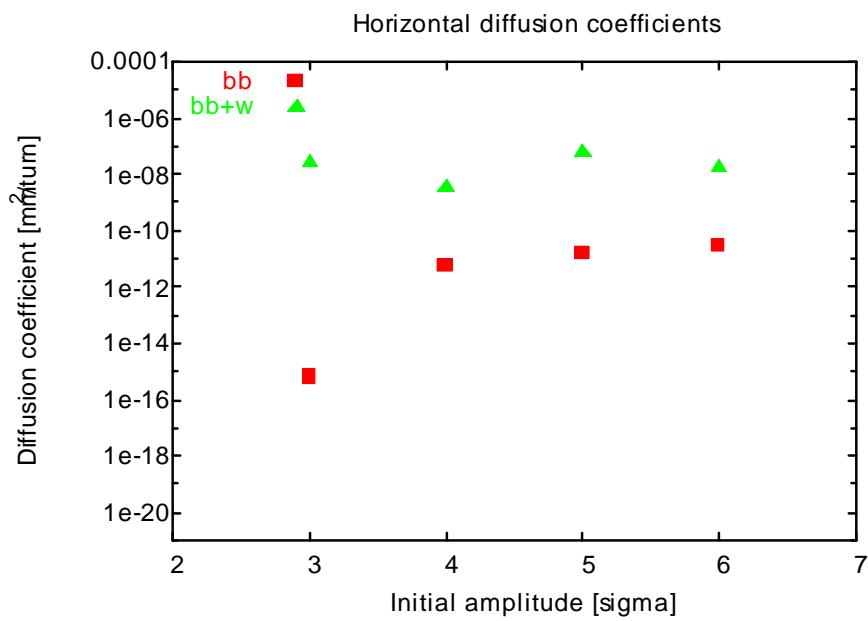


# Placing a wire next to the parasitic



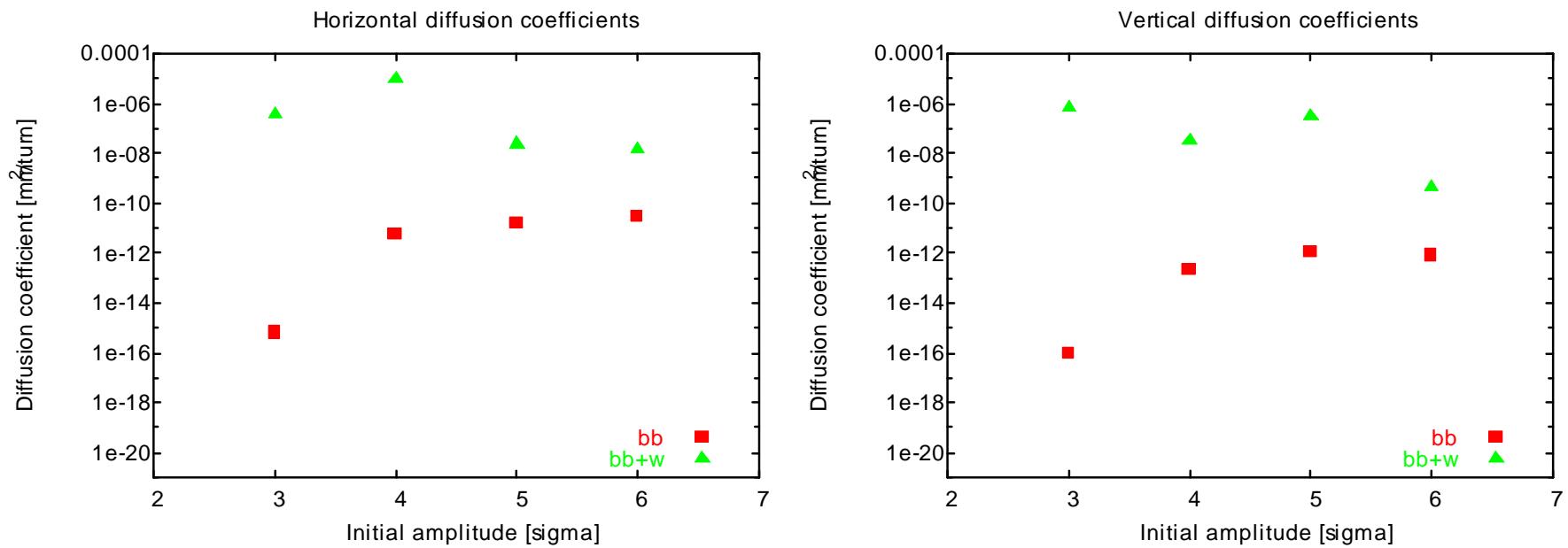
If beam has small (large) aspect ratio, the round beam correction principle does not work at this separation

# Placing a wire next to the parasitic and increasing the separation by 50 %



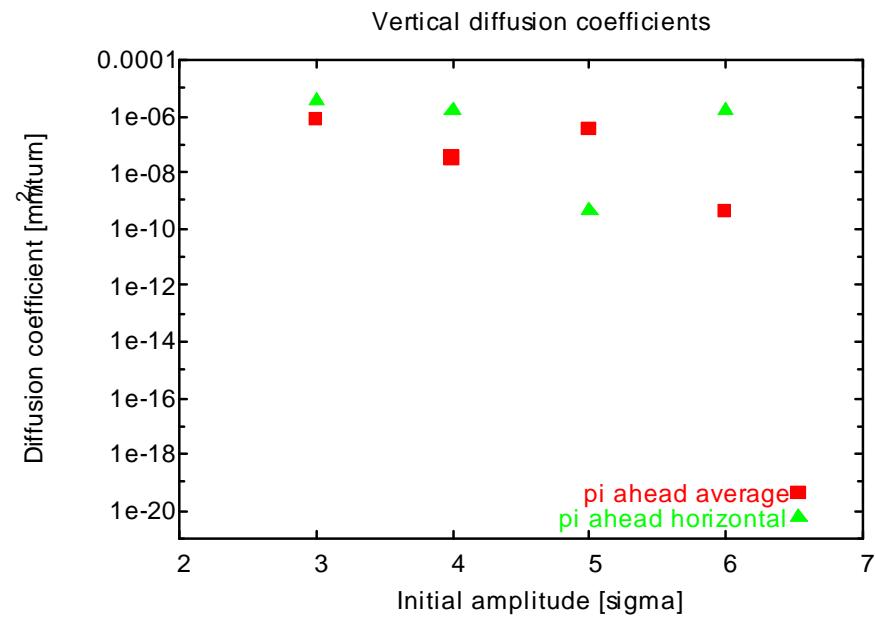
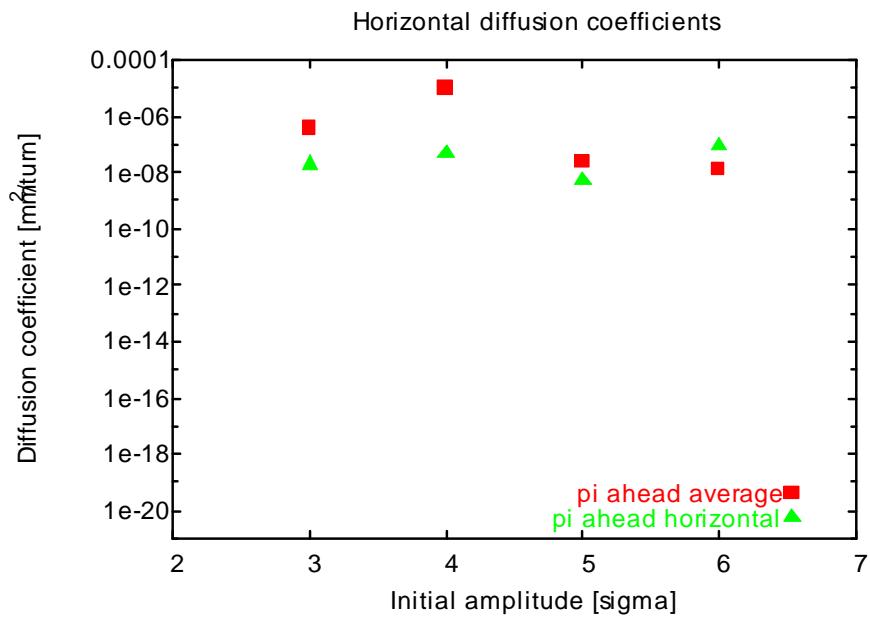
Increasing the separation by this amount still does not help the compensation as in the case of round beam-beam interactions

# Placing the wire $\pi$ ahead (average H & V)



Diffusion coefficients are even larger than when the wire was placed right next to the beam-beam interaction

# Placing the wire $\pi$ ahead horizontally



In general it helps to have the correct phase even in the non-round case

# Summary and plans

- Principle of (near) round beam-beam interaction compensation understood
- BBSIM passed all tests and checks, and confirmed what we expected from theory
- The principle does not work for beams with aspect ratios significantly different than 1 and relevant separations
- In general it is not possible to compensate exactly 2 (or more) round beam-beam kicks with a single wire
- Attempt to compensate the average kicks might not be sufficient
- Plan to try map minimization for the non-round case; hopefully will give more insight for further analytical work
- If it works, concentrate to collision, and the correction of the 4 nearest parasitics as a first step
- Further possibilities: different bunches, multiple wires, etc